SUST J Sci Tech, Vol 33(1) 2023, 1-4 ISSN- 2959-3522

Investigation on Opacity of Gaseous Stars

(Submitted: 26.10.2022 ; Accepted: 05.05.2023)

S.B. Faruque*, Sumit Roy Pronoy and Ahmed Al Imtiaz

Department of Physics, Shahjalal University of Science and Technology, Sylhet 3114, Bangladesh.

*Corresponding Author's E-mail: awsbf62@yahoo.com

Abstract

Following Sir A.S. Eddington's work on the interior of gaseous stars, we derive a simple equation that gives opacity of a star which depends on the mass of the star and another stellar parameter. We compare our result with the existing result of Kramer. We conclude that massive gaseous stars have lower density and higher temperature than low mass gaseous stars.

Key words: Gaseous star; mass-luminosity relation; opacity.

1. Introduction

The interior of a gaseous star was thoroughly investigated by Sir A. S. Eddington [1-2]. Luminosity and opacity were also investigated by E.A. Milne in 1937 [3]. But no precise equation is available till now except Kramer's law [4]. Opacity of a star is affected by many factors. Generally opacity is inverse of transparency. Lower opacity means higher transparency and vice versa. Moreover, metallicity affects opacity. Opacity of most main sequence stars is caused by bound -free and free-free transitions. For very hot stars it is due to electron scattering [5].

Opacity is an important stellar parameter and it affects stellar spectra. The classification of stellar spectra is an ongoing process. Consider a beam of light rays travelling through a gas. The change in intensity, dI_ν, of a ray of frequency ν as it travels through the gas is proportional to its intensity, I_ν, the distance traveled, ds, and the density of the gas, ϱ. That means [6]

$$dI_\nu = -\kappa_\nu \varrho I_\nu ds \quad (1)$$

The quantity κ_ν is called the absorption coefficient, or opacity, with the subscript ν indicating that opacity is frequency dependent. Opacity κ appears in energy transfer equation and represents the ability of stellar material to absorb radiation. To find opacity of stellar interior we need to consider all microscopic processes that can absorb photons at each frequency ν. There are four such processes and the corresponding opacities are labelled as $\kappa_{\nu,bb}$, $\kappa_{\nu,bf}$, $\kappa_{\nu,ff}$ and $\kappa_{\nu,es}$. In these symbols, the subscripts bf and ff stands for bound-free absorption and free-free absorption, respectively. And the subscripts $bb\ and\ es$ stands for bound-bound absorption and absorption by electron scattering, respectively.

A gaseous star of low density remains in hydrostatic equilibrium [2]. Eddington's investigation primarily was for gaseous stars. But he did not derive any opacity law or any kind of numerical work was done by him. In this work, we are going to find a simple method to find opacity of a gaseous star following Eddington's ideas of gaseous stars which he assumed to remain in thermodynamic equilibrium [2]. The paper is organized as follows: in Section 2, we derive a relation between opacity and mass of a star following Eddington's initial ideas [2] of the interior of a star. Finally in Section 3 we conclude.

2. Opacity of a Gaseous Star

If the chemical composition remains constant in time, a star remains in hydrostatic equilibrium following the equation

$$\frac{dP}{dr} = -g(r)\rho(r), \quad (2)$$

where $g(r)$ is the acceleration due to gravity at position r measured from the centre and $\rho(r)$ is the density at r. We have taken $g(r)$ as

$$g(r) = +\frac{GM(r)}{r^2}, \quad (3)$$

and $P(r)$ is the sum of thermodynamic pressure of an ideal gas, p_G and the pressure created by radiation, p_R. So,

$$P(r) = p_G + p_R. \tag{4}$$

For an ideal gas [2],

$$p_G = \frac{\mathfrak{R}}{m}\rho T. \tag{5}$$

And pressure due to radiation is

$$p_R = \frac{1}{3}\, aT^4 \tag{6}$$

where

$a = {}^{4\sigma}/_{c}$, and $\sigma = 5.67 \times 10^{-8}$ $\mathrm{Wm^{-2}K^{-4}}$. In Eqs.(5) and (6), T is the temperature, m is the molecular weight of the material inside the star and $\mathfrak{R}$ is the gas constant. Pressure due to radiation can be understood since radiation is forced through the matter.

If E is the outward flow of energy per square meter per second, then

$$E = constant \times \frac{1}{\kappa\rho}\frac{dp_R}{dr} = -\frac{c}{\kappa\rho}\frac{dp_R}{dr}, \tag{7}$$

where c is the speed of light in free space and κ is the opacity. Equation (7) is the equation of radiative equilibrium derived by Eddington [2,7].

Edddington [2] assumed that the total pressure P in the interior of a gaseous star is given by

$$P = \frac{p_G}{\beta}, \tag{8}$$

so that

$$p_R = (1-\beta)P. \tag{9}$$

From Eqs. (5) and (6), we obtain

$$P = \frac{1}{3}\frac{a}{1-\beta}T^4. \tag{10}$$

The term β is a constant characteristic of a star and defined through Eqs.(8) and (9). Equation (8) can be written as

$$P = \frac{\mathfrak{R}}{\beta m}\rho T. \tag{11}$$

From Eq. (10), we obtain

$$T = \left(\frac{3(1-\beta)}{a}P\right)^{1/4}. \tag{12}$$

From Eqs.(2) and (7), we can eliminate dr and obtain

$$dP = \frac{g}{E}\frac{c}{\kappa}dp_R\,. \tag{13}$$

Eddington assumed that κ is constant throughout the star. He also assumed that E is proportional to g throughout the star. Subsequently, he assumed

$$\frac{E\kappa}{gc} = 1-\beta, \tag{14}$$

which is a constant for the star in question. If L is the luminosity, M is the mass, R is the radius and G, the constant of gravity, then at the boundary of the star,

$$L = 4\pi R^2 E,$$

and

$$g = \frac{GM}{R^2}\,.$$

Then, Eq.(14) gives

$$\frac{L\kappa}{4\pi GcM} = 1-\beta,$$

or

$$L = \frac{4\pi Gc}{\kappa}M(1-\beta). \tag{15}$$

Rearranging Eq.(15), we can write

$$M = \beta M + \frac{\kappa}{4\pi Gc}L\,. \tag{16}$$

At this stage, we digress from Eddington's work. We write

$$M = qM_\odot\,, \tag{17}$$

where q is a constant and $M_\odot$ is the mass of sun. Using the mass-luminosity relation [8],

$$\frac{L}{L_\odot} \approx 1.4\left(\frac{M}{M_\odot}\right)^{3.5}, \quad \text{for } 2M_\odot < M < 55M_\odot\,. \tag{18}$$

Equation (16) can be rearranged to give

$$q = q\beta + \frac{\kappa}{4\pi Gc}\frac{L}{M_\odot}\,. \tag{19}$$

Using Eq.(18), Eq. (19) can be reduced to

$$q^{3.5} = q(1-\beta)\frac{4\pi Gc}{1.4\kappa}\left(\frac{M_\odot}{L_\odot}\right). \tag{20}$$

Equation (20) gives

$$q^{2.5} = 898 \times \frac{1-\beta}{\kappa}, \tag{21}$$

in which we have used the standard values of mass and luminosity of the sun. Now, the Kramer's law gives [4]

$$\kappa = \kappa_0 \rho T^{-3.5}. \quad (22)$$

In Eq.(22), $\kappa_{\nu,bf}$ and $\kappa_{\nu,ff}$ contributions dominate.

In Figure 1, we plot κ versus$(1-\beta)$, for different values of q found from Eq.(21).. The figure clearly shows that as q increases κ decreases and as β decreases κ increases.

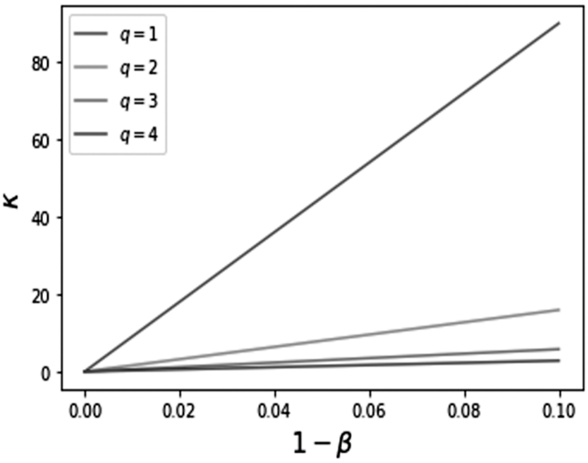

Fig. 1: κ versus $1-\beta$ graph

In our investigation, we did not enter into the mechanisms responsible for opacity. We used Eddington's study and arrived at an equation (Eq.(21)) that gives opacity in terms of mass of a star. But since opacity decreases with increasing mass, we conclude that as mass increases the density of a gaseous star decreases and temperature increases. Only this way opacity can decrease as mass increases. The dependence of opacity on ? is due to energy flow outward in the interior of a star and cannot be quantified easily. In short, we have done an investigation on opacity that depends on mass and luminosity of a gaseous star. Our work is based on Eddington's study of stars. Eddington investigated also the physics of white dwarf stars [9-10] and those works require further attention to find new insight on white dwarf matter.

3. Summary

We have studied physics of the interior of gaseous stars. Star's interior contains mainly hydrogen, helium and elements heavier than helium. These heavier elements are collectively called metals and thus the term metallicity arises. Metallicity i.e. massive elements affect opacity of a star. The lower the opacity, the greater is the transparency. We have found that opacity decreases as mass of a star increases. So, we conclude that massive gaseous stars have lower density and higher temperature. Also, careful analysis of Eq. (21) and Eq. (22) reveal that for stars with volume equal to the volume of the sun, β is proportional to the density ϱ of the star and is also proportional to $1/T^{3.5}$. Moreover, for all stars, β is a unique thermodynamic parameter characteristic of a star. We have found that β and κ are closely connected as revealed in Figure 1. As a conclusion, we can say that our analysis revealed for the first time that opacity is related to the stellar parameter β through Eq.(21) and this is simpler than the relation given in Eq. (14). Moreover, from mass-luminosity relation one can find mass by measuring luminosity. That mass used in Eq. (21) gives $(1-\beta)/\kappa$. Using this quantity in Eq.(14), we can find E. Thus, Eq.(14) and Eq.(21) together gives a newer way to find energy radiated by a star per unit area per unit time. Hence, we have found new physics applicable to gaseous stars.

Acknowledgement

We gratefully acknowledge the financial grant from Shahjalal University of Science and Technology's Research Centre for the present work.

References:

1. Eddington, A.S. (1916). On the radiation equilibrium of the stars, Mon. Not. R. Astron. Soc., 77, 16-35.
2. Eddington, A.S. (1918). On the conditions in the interior of a star, ApJ, 48, 205-213.
3. Milne, E.A., (1937). Stellar luminosity and the opacity in the outer layers of a star, Mon. Not. R. Astron. Soc., 98, 21-41.
4. Choudhury, A.R. (2010). Astrophysics for Physicists, Cambridge U. Press,
5. Chowdhury, N.M. & Hossain, M.M. (2012). A study of a stellar model with Kramer's opacity and negligible abundance of heavy elements, J. Appl. Comput. Math. 1, 1000103.
6. B.W. Caroll and D.A. Ostlie (2017). An Introduction to Modern Astrophysics, Cambridge U. Press.

7. Crothers, S.J. & Robitaille, P-M. (2019). Eddington's mass-luminosity relations and the laws of thermodynamics, Physics Essays 32, 353-357.

8. Kuiper, G.P. (1938). The empirical mass -luminosity relationship, ApJ, 88, 472-507.

9. Eddington, A.S. (1935). The pressure of a degenerate gas and related problems, Proc. R. Soc. London A, 152, 253-272.

10. Edddington, A.S. (1940). The physics of white dwarf matters, Mon. Not. R. Astron. Soc., 100, 582-594.