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Gravitomagnetic Time Delay for Light Propagating through a Rotating Thin Shell

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Abstract

When a light ray passes through a slowly rotating thin shell of matter as well as moving in the exterior field of slowly rotating distant body, then the time of passage of the ray through the shell suffers a time delay which is called the gravitomagnetic time delay. In this article, we devise a simplified configuration of the problem where the light ray moves along a line that is parallel to the spin direction of the distant body and at the same time parallel to the axis of the thin shell but the ray does not go through the center of the thin shell. We calculate both the prograde and retrograde time delay of the light ray and find that gravitomagnetic time delay is positive for the prograde motion and negative for the retrograde motion of the light ray. That means the prograde motion takes longer time than the retrograde motion to cover the same distance. The result is analogous to that which happens in gravitomagnetic clock effect (GCE); the difference being that the motion in time delay considered here is straight line motion whence in GCE the point mass move along circular path around a rotating source. Although gravitomagnetic time delay is an old problem but the present work shed new light on this problem.

1. Introduction

Gravitoelectromagnetism (GEM) is a set of formal analogies between the equations of electromagnetism and relativistic gravitation. Gravitomagnetism is part of GEM and is the kinetic effects of gravity, in analogy to the magnetic effects of moving electric charge [1].

There are many consequences of gravitomagnetism such as gravitomagnetic clock effect (GCE) [1, 7, 8, 11], the Lense–Thirring effect [9], gravitomagnetic time delay [2, 3, 4, 5, 6], Sagnac effect [10]. The gravitomagnetic time delay appears when a light ray crosses a slowly rotating thin shell of matter as well as propagating in the exterior field of a distant rotating astronomical body [2, 3, 4, 5, 6]. In the present work, we consider a special configuration of the problem and derive the gravitomagnetic time delay that follows. The problem is devised in Section 2, the derivation of the result is done in Section 3, and in Section 4, we conclude.

2. Gravitomagnetic time delay: The problem

We consider a slowly rotating astronomical body whose equatorial plane is the x–y Cartesian plane. If Φ_g is the gravitational potential and $\vec{A}_g$ is the gravitomagnetic vector potential created by the source, then far from the source

$$\Phi_g \sim \frac{GM}{r}, \qquad \vec{A}_g \sim \frac{G\vec{J}\times\vec{r}}{c\,r^3}, \tag{1}$$

where M and $\vec{J}$ are the total mass and angular momentum of the source. Let Δ denote the time delay of a light ray compared to the time it takes to pass the shell according to the flat metric.

For the case of a light ray crossing the thin shell as well as moving in the fields stated above, we shall calculate the time delay between the point through which the ray enters the shell, namely $P_1 : (ct_1, \vec{X}_1)$, and the point the ray leaves the shell, $P_2 : (ct_2, \vec{X}_2)$.

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3. Calculation of time delay

Let the origin of the Cartesian co-ordinate system we use be the center of the rotating distant source body. We choose, $P_1 \equiv (ct_1, -x_1, y_1, z_1)$ and $P_2 \equiv (ct_2, x_2, y_2, -z_2)$ for retrograde motion of the light ray when the angular momentum of the distant source is $\vec{J} = J\hat{k}$. For prograde motion of the ray $P_1 \equiv (ct_1, x_2, y_2, -z_2)$ and $P_2 \equiv (ct_2, -x_1, y_1, z_1)$. And $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Furthermore, the light ray propagates in the retrograde sense (i.e. along $-\hat{k}$) in the first case and in the second case (prograde sense) it is along $\hat{k}$. Now,

$$\vec{J} \times \vec{r} = J(-y\hat{i} + x\hat{j}), \tag{1}$$

and,

$$\vec{A}_g = \frac{GJ}{c}\frac{(-y\hat{i} + x\hat{j})}{r^3}, \tag{2}$$

$$\vec{A}_g \cdot d\vec{X} = -\frac{GJ}{r^3 c}(y\,dx - x\,dy). \tag{3}$$

It is shown in Ref. [2] that gravitomagnetic time delay is given by,

$$\Delta_{GM} = -\frac{2}{c^3}\int_{P_1}^{P_2} \vec{A}_g \cdot d\vec{X}. \tag{4}$$

Using Eq. (3), we find in our case

$$\Delta_{GM} = \frac{2GJ}{c^4}\left[\int_{P_1}^{P_2} \frac{(y\,dx - x\,dy)}{r^3}\right]. \tag{5}$$

Using the definition of P_1 and P_2 in Eq. (5), we find

$$\Delta_{GM} = \frac{2GJ}{c^4}\left[\frac{y_2 - x_2}{r_2^3} - \frac{y_1 + x_1}{r_1^3} + \frac{1}{r_2}\left(\frac{1}{y_2} - \frac{1}{x_2}\right) - \frac{1}{r_1}\left(\frac{1}{y_1} + \frac{1}{x_1}\right)\right], \tag{6}$$

for light ray propagating in the retrograde sense. Now, without loss of generality, we can set $x_1 = y_1$ and $x_2 = y_2$. Then, gravitomagnetic time delay for the retrograde motion of a light ray in this special case becomes

$$\Delta_{GM} = \frac{2GJ}{c^4}\left[-\frac{x_1 + y_1}{r_1^3} - \frac{2}{r_1 y_1}\right], \tag{7}$$

where $r_1^2 = x_1^2 + y_1^2 + z_1^2$ and $r_2^2 = x_2^2 + y_2^2 + z_2^2$. Inspection of Eq. (7) clearly shows that Δ_{GM} for retrograde motion of a light ray is negative.

For prograde motion of a light ray in the present configuration is

$$\Delta_{GM} = \frac{2GJ}{c^4}\left[\frac{x_1 + y_1}{r_1^3} + \frac{2}{r_1 y_1}\right]. \tag{8}$$

The delay (8) is definitely positive. Thus, for prograde motion the sign of gravitomagnetic time delay is positive and that for retrograde motion it is negative.

4. Conclusion

We have considered a light ray that propagates in the exterior field of a slowly rotating distant massive body and passes through a slowly rotating thin shell of matter. The angular momentum of the distant body is along the $+z$-axis and the light ray moves in a straight line entering the shell at the point $P_1 : (ct_1, \vec{X}_1)$ and leaves the shell from the point $P_2 : (ct_2, \vec{X}_2)$. The light ray moves within the shell along a straight line that is parallel to the axis of the shell, i.e. it does not pass through the center of the shell. If it did so then time delay would have been zero.

The light ray's path is prograde when it moves along $+z$-axis and the path is retrograde when it moves in the opposite direction. We chose simple geometry of the passage of the ray, i.e. we have used Cartesian co-ordinates to specify spatial entry and exit points for both the senses of motion of the light ray. The gravitomagnetic time delay which we have found is positive for prograde and negative for retrograde motion of the light ray. That means the ray takes longer time for passage through the shell when it moves along the spin direction of the distant source body (prograde motion) and less time for retrograde path.

This result is analogous to what we find in gravitomagnetic clock effect (GCE). So, we have found another clock effect that occurs for straight line motion instead of circular motion of GCE. The result is interesting and warrants further investigation. Gravitomagnetic time delay is a topic of gravitomagnetism. Once observed, this will confirm the existence of magnetism in the realm of gravitation. So, this effect is crucial to complete the theory of gravitation. Potential applications of gravitomagnetic time delay include space missions, especially in the navigation of spacecraft. Future work on this problem can be on the construction and installation of rotating thin shells on spacecraft that move near spinning stars, Kerr black holes, etc. In conclusion, we have added a piece of new information to the existing literature.

References

[1] B. K. Chakravorti and S. B. Faruque, "Gravitoelectromagnetism and gravitomagnetic clock effect," *J. Sci. Res.*, vol. 14, no. 3, pp. 749–754, 2022. https://doi.org/10.3329/jsr.v14i3.57972.

[2] I. Ciufolini *et al.*, "On the gravitomagnetic time delay," *Phys. Lett. A*, vol. 308, no. 2–3, pp. 101–109, 2003. https://doi.org/10.1016/s0735-9601(02)01804-2.

[3] M. Sereno, "On gravitomagnetic time delay by extended lenses," *Mon. Not. R. Astron. Soc.*, vol. 357, no. 4, pp. 1205–1210, 2005. https://doi.org/10.1111/j.1365-2966.2005.08709.x.

[4] I. Ciufolini and F. Ricci, "Time delay due to spin and gravitational lensing," *Class. Quantum Grav.*, vol. 19, no. 15, pp. 3863–3874, 2002. https://doi.org/10.1088/0264-9381/19/15/301.

[5] L. Iorio, "Light deflection and time delay in the gravitational field of a spinning body," *Il Nuovo Cimento B*, vol. 118, no. 3, pp. 249–258, 2003.

[6] S. M. Kopeikin and E. B. Formalont, "Gravitomagnetism, causality, and aberration of gravity in the gravitational light-ray experiments," *Gen. Rel. Grav.*, vol. 39, no. 10, pp. 1583–1624, 2007.

[7] J. M. Cohen and B. Mashhoon, "Standard clocks, interferometry, and gravitomagnetism," *Phys. Lett. A*, vol. 181, no. 5, pp. 353–358, 1993.

[8] S. B. Faruque, "Gravitomagnetic clock effect in the orbit of a spinning particle orbiting the Kerr black hole," *Phys. Lett. A*, vol. 327, no. 2–3, pp. 95–97, 2004. https://doi.org/10.1016/j.physleta.2004.05.018.

[9] H. Thirring and J. Lense, "On the influence of the proper rotation of a central body on the motion of the planets and the moon according to Einstein's theory of gravitation," *Zeit. Phys.*, vol. 19, pp. 156–193, 1918.

[10] A. Tartaglia and M. L. Ruggiero, "The Sagnac effect and pure geometry," *Am. J. Phys.*, vol. 83, pp. 427–432, 2015. https://doi.org/10.1119/1.4904319.

[11] S. B. Faruque, "Clock effect due to gravitational spin-orbit coupling," *Phys. Lett. A*, vol. 359, no. 4, pp. 252–255, 2006. https://doi.org/10.1016/j.physleta.2006.06.046.

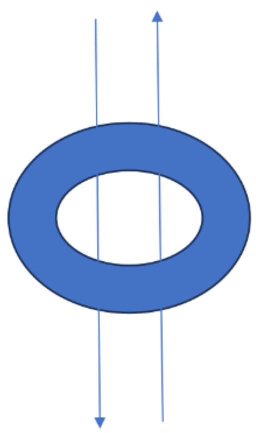

Figure 1. Light rays (prograde and retrograde) passing through the rotating thin shell.