Rest Mass and Gravitational Mass

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Abstract

A satisfactory theory of quantum gravity requires a quantum formulation of the classical equivalence principle. Recently, a formulation of the quantum equivalence principle (QEP) appeared in terms of the equality of rest, inertial, and gravitational masses, which is valid for non-relativistic particles in a weak gravitational field. In this work, we employ a fully general relativistic Schwarzschild metric assuming unequal rest and gravitational masses. Thereafter, we derive a relation between the rest mass and the gravitational mass of a particle. It is shown that in the absence of gravity, the gravitational mass becomes the relativistic mass.

Keywords: Quantum equivalence principle; Schwarzschild metric; Gravitational mass; Relativistic mass.

1. Introduction

The equivalence principle is the heart of gravitational physics [1-3]. It says that the inertial mass and the gravitational mass are equal. On the other hand, gravity is expected to be a quantum force. Therefore, there must exist a QEP and it is argued that QEP should be exactly satisfied in nature [4, 5]. Thus, we encounter three masses of a particle i.e. m_I, m_R, and m_G, where I, R, and G stand for inertia, rest, and gravity, respectively. It is shown by Das *et al.* [4] that the Schwarzschild metric in the coordinates (t, r, θ, φ) gets modified when the three masses are not equal, and it then reads

$$dS^2 = c^2 dt^2 \left(1 - \frac{m_G}{m_R}\frac{a}{r}\right) - \frac{dr^2}{1 - \frac{m_G}{m_R}\frac{a}{r}} - r^2(d\theta^2 + sin^2\theta\, d\varphi^2) \quad (1)$$

where the Schwarzschild radius, $a = 2GM/c^2$ (G is the universal gravitational constant and M is the mass associated with the static spherically symmetric metric). Our aim in this work is to find a relation between m_G, and m_R. Section 2 provides the calculation and result of this study and finally, the conclusion is arrived at in Section 3.

2. Calculation and Result

We have in special relativity the time dilation formula

$$T = \gamma\, T_0 \quad (2)$$

where T_0 is the proper time in frame S' and T is the coordinate time in frame S, and

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad (3)$$

Therefore, velocity influences time. We know that mass also influences time. Thus, in our endeavor γ will be modified to incorporate mass.

Let us consider timekeeping in an earthbound laboratory on the equator [6]. This entails the following relations:

$$t = \gamma\tau,\ r = \text{constant},\ \theta = \frac{\pi}{2},\ \text{and}\ \varphi = \omega\tau.$$

Here, τ is the proper time and the polar spherical angular coordinates: θ is called the colatitude and φ is called the longitude.
The four-velocity is then

$$\dot{x}^\mu = (\gamma, 0, 0, \omega) \quad (4)$$

Equation (1) in this case gives

$$c^2 = g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = c^2\left(1 - \beta\frac{a}{r}\right)\gamma^2 - r^2\omega^2 \quad (5)$$

where $\beta = \frac{m_G}{m_R}$. Moreover, a distant observer records the velocity of the laboratory as

$$v = r\frac{d\varphi}{dt} = \frac{r\omega}{\gamma} \quad (6)$$

which gives $\omega = \frac{\gamma v}{r}$. Eliminating ω from the two

https://doi.org/10.63512/sjst.2024.1001
Received 11 November 2024, Accepted 14 May 2025.

previous equations, we get

$$\gamma = \left[1 - \left\{\beta \frac{a}{r} + \frac{v^2}{c^2}\right\}\right]^{-\frac{1}{2}} \quad (7)$$

Equation (1) holds that each gravitational mass and relativistic mass are different, but if they are same, we should recover the Schwarzschild metric from Eq. (1). However, we have $m_G = \beta m_R$ whence for a particular velocity v, $m_G \propto m_R$. Since, relativistic mass m_R is influenced by velocity as $m = m_0\gamma$, m_G would be influenced by velocity in the same way since they are proportional. Thus, we can write

$$m_G = \frac{m_R}{\sqrt{1-\left(\frac{v^2}{c^2}+\beta\frac{a}{r}\right)}} \quad (8)$$

where β is a positive quantity. Therefore, the way v influences mass, $\beta\frac{a}{r}$ influences mass in the same way.

We can rearrange Eq. (8) to find an expression for β. Dividing Eq. (8) by m_R we obtain, to first order in a,

$$\beta = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{\beta}{2}\frac{a}{r}$$

which yields,

$$\beta\left(1 - \frac{a}{2r}\right) = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\text{Or} \quad \beta\left(1 + \frac{a}{2r}\right)^{-1} = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad (9)$$

Therefore, we obtain an approximate value for β for all velocities v, as

$$\beta = \frac{1+\frac{a}{2r}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1+\frac{GM}{c^2 r}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (10)$$

If the source of gravity is absent, Eq. (10) reduces to

$$m_G = \frac{m_R}{\sqrt{1-\frac{v^2}{c^2}}} \quad (11)$$

Hence, in the absence of gravity, m_G is the relativistic mass m and m_R is the rest mass m_0. Thus Eq. (11) gives the well-known relation

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (12)$$

and Eq. (10) can be written as

$$m_G = \frac{m_0\left(1+\frac{GM}{c^2 r}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \quad (13)$$

Eq. (13) can be written more clearly as

$$m_G = \frac{m_0\left(1-\frac{\Phi}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} \quad (14)$$

where $\Phi = -\frac{GM}{r}$, the Newtonian gravitational potential at r. If $\Phi = 0$, $m_G = m$, the relativistic mass. The asymptotic form of Eq. (14) is the well-known formula, Eq. (12).

3. Conclusion

If QEP holds exactly, the theory of gravity would involve unequal inertial, gravitational, and rest masses. In the present work, we have derived a relation between the gravitational and rest masses of a body (Eq. 8). If $\frac{m_G}{m_R} = \beta$, then β is a dynamical quantity that takes values from minimum to greater values, which is obvious from Eq. (10). As far as our knowledge, this is the first time a relation between the gravitational and rest masses of a body is found. This relation may serve as a basis for developing theoretical constructions in quantum gravity. As a particle moves its mass increases; similarly, in a gravitational field, the particle gains speed toward the source of gravity. Therefore, its mass should increase and this is shown in Eq. (14). We hope our work will inspire to explore quantum nature of gravitational mass and could motivate deeper investigations into the mass-energy relationship in gravity.

4. Acknowledgement

The authors would like to extend their gratitude to the Astronomy and Astrophysics Laboratory at the Department of Physics, Shahjalal University and Technology, for providing invaluable resources and support throughout the course of this research.

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