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Sagnac Effect with Gravity Correction

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ABSTRACT

Sagnac was an advocate of aether theory. He did experiment to prove the existence of aether. His experiments used two light rays moving in opposite directions along a circular ring that is rotating with angular velocity ω. Theoretically, such rays will take different times to arrive at the endpoint, and this theoretical fact was demonstrated by Sagnac himself and by many other experimenters. The usual Sagnac effect is only affected by ω; the gravitational field's effect is absent. In this work, we correct the Sagnac time delay by considering the influence of gravity on time. Our formula reduces to the usual one when the Schwarzschild radius is very small.

1. Introduction

In 1899, G. Sagnac developed a theory regarding the existence of a motionless aether [1]. His aim was to explain all optical phenomena within this framework, giving particular attention to the Fresnel–Fizeau experiment [2, 3]. The Sagnac experiments have many different alternative explanations in the framework of general relativity [4]. In this work, we take a look at this effect, i.e, we compute the difference in time to travel the same loop, which is rotating, by two counter-rotating light rays, and find the influence of gravity on the time difference. In Section 2, we work out the main effect, and in Section 3, we conclude.

2. Sagnac Effect with Gravity Correction

We consider two light rays in opposite directions around a static circular ring of radius r. The two rays will arrive at the end point simultaneously. But if the ring is rotating, the ray that travels in the same direction as the rotation of the ring travels a distance greater than that of the ray traveling in the opposite direction.

So, the counter-rotating ray will arrive earlier than the co-rotating ray. The length of the path is $2\pi r$ and if there is no angular velocity of the loop, the duration of the path the ray take is:

$$\Delta t_0 = \frac{2\pi r}{c} \tag{1}$$

If there is a loop rotation with angular velocity, we shall have to use general relativity to find Δt_0 (Eq. 1). In an Earth Centered Inertial frame (ECI), the spacetime is flat i.e. Minkowskian and in cylindrical coordinates, the metric is:

$$\begin{aligned} ds^2 &= c^2dt^2 - dr^2 - r^2d\phi^2 - dz^2 \\ &= c^2dt^2 - d\sigma^2 \end{aligned} \tag{2}$$

where, $d\sigma^2 = dr^2 + r^2d\phi^2 + dz^2$ is the squared coordinate distance.

Since the loop is rotating with angular velocity ω, the coordinates we have to use are the transformed (t', r', ϕ', z'), and these are given by

$$t = t', \quad r = r', \quad \phi = \phi' + \omega t, \quad z = z' \tag{3}$$

which an observer fixed on the rotating loop's frame finds. Then, the metric (2) becomes

$$\begin{aligned} ds^2 &= c^2dt^2 - dr^2 - r^2\left(d\phi' + \omega\, dt\right)^2 - dz^2 \\ &= c^2dt^2 - dr^2 - r^2d\phi'^2 - 2\omega r^2 d\phi'\, dt \\ &\quad - r^2\omega^2dt^2 - dz^2 \end{aligned}$$

Or,

$$\begin{aligned} c^2\left(1 - \frac{r^2\omega^2}{c^2}\right)dt^2 - 2\omega r^2 d\phi'\, dt \\ - \left(dr^2 + r^2d\phi'^2 + dz^2\right) = 0 \end{aligned} \tag{4}$$

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for light rays. The spatial metric becomes, with $dr = dz = 0$

$$dL^2 = \frac{r^2 d\phi'^2}{1 - \frac{r^2\omega^2}{c^2}}$$

from which we obtain

$$dL = \frac{r\, d\phi'}{\sqrt{1 - \frac{r^2\omega^2}{c^2}}} \tag{5}$$

The path of the light rays is given by

$$\begin{aligned} L &= \int_0^{2\pi} \frac{r\, d\phi'}{\sqrt{1 - \frac{r^2\omega^2}{c^2}}} \\ &= \frac{2\pi r}{\sqrt{1 - \frac{r^2\omega^2}{c^2}}} \end{aligned} \tag{6}$$

Now, the speed of the light ray in a co-rotating motion appears as

$$v_+ = c - r\omega \tag{7}$$

and that in counter-rotating motion appears as

$$v_- = c + r\omega \tag{8}$$

The duration in the co-rotating motion is given by

$$t_+ = \frac{L}{v_+} \tag{9}$$

and that in counter-rotating motion is

$$t_- = \frac{L}{v_-} \tag{10}$$

The difference in journey times of the two rays is

$$\begin{aligned} \Delta t_0 = t_+ - t_- &= \frac{L}{v_+} - \frac{L}{v_-} \\ &= \frac{Lv_- - Lv_+}{v_+ v_-} \end{aligned} \tag{11}$$

So,

$$\Delta t_0 = \frac{2\pi r(c + r\omega) - 2\pi r(c - r\omega)}{\sqrt{1 - \frac{\omega^2 r^2}{c^2}}\ (c^2 - r^2\omega^2)} \tag{12}$$

which reduces to,

$$\Delta t_0 = \frac{4\pi r^2\omega}{c^2\left(1 - \frac{r^2\omega^2}{c^2}\right)\sqrt{1 - \frac{\omega^2 r^2}{c^2}}} \tag{13}$$

Equation (13) gives the exact Sagnac time delay in the proper frame of the rotating ring.

However, this is the proper time in the frame of reference of the rotating ring. A stationary observer a long way off will measure the velocity of the rotating ring as v, and the time will appear as dilated by a factor of γ given by [5]

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2} + \frac{a}{r}\right)}} \tag{14}$$

where, $a = \frac{2GM_E}{c^2}$ is the Schwarzschild radius with M_E as the mass of Earth.

So, in reality, the Sagnac time delay would appear as

$$\Delta t = \frac{4\pi r^2\omega}{c^2\left(1 - \frac{r^2\omega^2}{c^2}\right)^{3/2}} \frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2} + \frac{a}{r}\right)}} \tag{15}$$

But since $v \ll c$ and also $\omega^2 r^2 \ll c^2$, We will have the approximate formula for the Sagnac effect as follows:

$$\Delta t \cong \frac{4\pi r^2\omega}{c^2\sqrt{1 - a/r}} \tag{16}$$

But for cases where $\frac{a}{r} \ll 1$, We will get

$$\Delta t = \frac{4\pi r^2\omega}{c^2} \tag{17}$$

which is the approximate formula found by many authors (see e.g., [4] and the references therein).

3. Conclusion

In this paper, we have calculated the Sagnac time delay taking into account the Earth's gravitational field. Our result (Eq. 15) slightly differs from the usual result (Eq. 17) [4].

However, when $\omega^2 r^2 \ll c^2$, and $v \ll c$, we obtain a gravity corrected formula for Sagnac effect given by Eq. (16). When $a/r \ll 1$, the original formula (Eq. 17) is correct practically. However, if $\frac{a}{r}$ cannot be neglected, i.e., $a/r \approx 1$, our computed formula will work well to determine Sagnac time delay. In conclusion, we have revisited the Sagnac effect and calculated the contribution of Earth's gravitational field on the Sagnac time delay. Earth's gravitational field significantly affects the Sagnac time delay and it increases the time delay from the usual delay where Earth's gravity field is ignored. Hence, our work warrants attention and revision by researchers.

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