An Alternative Approach to Gravity based on Multiple Independent Fields

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Abstract

We present a new approach to gravity theory. In this approach, four independent fields convey the gravitational force. These fields are colored in the sense that they do not add up to Newtonian potential and field for two body gravitational interaction. The fields are generated by two types of mass of a single body; the gravitoelectric charge (mass)m and gravitomagnetic charge (mass) $-2m$. Here, there is no disparity between source and test particle masses which is usually seen in typical gravitoelectromagnetic theory. We also show that a wave picture can be generated from our theory and these waves are similar to that of general relativity, that is, the radiation is quadrupolar. Thus, a new paradigm of gravity theory has been opened up in this paper.

Keywords: Gravity, Gravitoelelctromagnetism, Maxwell's electromagnetic equations, Gravitational wave.

1. Introduction

The field equations of gravity resemble those of electrodynamics for slowly moving gravitational sources [1]. This fact gave rise to the theory of gravito-electromagnetism [2,3] which stands in between Newtonian gravity theory and the General Relativity. In this theory, the electro dynamical formulae are applicable. Hence, a source of gravity of mass M is treated as though it has gravito-electric charge of M and gravitomagnetic charge of $2M$. On the other hand, a test body of Newtonian mass m is treated as though it has gravito-electric charge of $-m$ and the corresponding gravito-magnetic charge is $-2m$ [4-5]. This disparity between source body and test body has been applied to ensure that the force is always attractive [4-5]. This is superficially justified, but conceptually unjustified, because, so far no evidence of difference in what we call mass is found. In the present paper, we use modified gravito-magnetism in a way to rediscover Newtonian gravity with four independent fields and extend that to formulate Maxwellian equations following the work of Faruque [6]. In the next section, we summarize the work of Faruque [6], then in section 3, we present the Maxwellian equations. In section 4, we calculate the influence of gravitational wave made through the Maxwellian equations on a single particle. We will see that, it is possible to construct real gravitational wave proved through the trajectory of particle on which the wave acts. Finally, in section 5, we conclude our work.

2. Multiple Independent Fields and Gravity

As depicted in Ref. [6], we associate with every material body a direct or so called electric mass m and a dual or so called magnetic mass$(-2m)$. A source of gravitational field i.e., a body of mass M has gravito-electric mass M and a gravito-magnetic mass$(-2M)$. In the same fashion the masses of the particle with which the gravitational field of the above body interacts, possess a gravitoelectric mass m and a gravitomagnetic mass$(-2m)$. This is because gravitation behaves in the same way irrespective of the particle being the source of field or the one of the field is forcing upon. The source body of electric mass M and magnetic mass $(-2M)$ produces four color fields, two of them are converging to the source and two diverging from the source. We take these fields by proposition as follows: (In what follows the gravitational constant G is taken as unity),

$$\vec{E}_1 = \vec{E}^{ee} = \frac{M}{r^2}\hat{r}, \tag{1}$$

$$\vec{E}_2 = \vec{E}^{bb} = -\frac{2M}{r^2}\hat{r}, \tag{2}$$

$$\vec{E}_3 = \vec{E}^{eb} = \frac{3}{2}\frac{M}{r^2}\hat{r}, \tag{3}$$

$$\vec{E}_4 = \vec{E}^{be} = -\frac{3M}{r^2}\hat{r}. \quad (4)$$

where the source body of gravito-electric charge M (ordinarily known as the mass) is assumed to be located at the origin of coordinate system and r is the radial coordinate. Here, in $\vec{E}^{ij}$, the superscript $i(\equiv e,b)$ refers to the source field which when i=e, it is electric and when $i=b$, it is magnetic. The second superscript $j(\equiv e,b)$ refers to which of the charges of a test body it will interact. If $j=e$, the field $\vec{E}^{ee}$ will act on the gravito-electric charge $q_E = m$ of the test body. When $j=b$, the field, say, $\vec{E}^{eb}$ will act on the gravitomagnetic charge of the test body which is $q_B = -2m$.

The characteristics of the fields (1) to (4), as evident from the expressions, are such that gravitoelectric charge $q_E = M$ produce diverging fields and gravitomagnetic charge $q_B = -2M$ produce converging fields. However, the cross fields are $\frac{3}{2}$ times stronger than the direct fields. The test particle of gravitoelectric charge m and gravitomagnetic charge $(-2m)$ experience four forces given by the symmetrical electrodynamic prescription,

$$\vec{F} = q\vec{E}\ . \quad (5)$$

We note that

$$\vec{E}_{resultant} = \vec{E}^{ee} + \vec{E}^{bb} + \vec{E}^{eb} + \vec{E}^{be} = -\frac{5}{2}\frac{M}{r^2}\hat{r} \neq \vec{E}_{Newtonian} \quad (6)$$

However $\vec{F}_{resultant} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = -\frac{Mm}{r^2}\hat{r} = \vec{F}_{Newtonian}$ (7)

Hence the fields $\vec{E}^{ee}$, $\vec{E}^{bb}$etc. are colored; they do not add up to the Newtonian field, but the forces on the gravitoelectric and gravitomagnetic charges of the test particle due to the four color fields add up to the net force as equal to what we know as the Newtonian gravitational force.

The sum of four potentials is

The resulting potential energies of the test particle is given by the prescription $\vec{F} = -\vec{\nabla}U$, U being the potential energy,

$$U_{resultant} = -\frac{Mm}{r} = U_{Newtonian}. \quad (8)$$

Hence, like the force, the potential energies are colorless and add up to yield the correct resultant as given by the Newtonian potential energy.

The four potentials in which the test particle finds itself are given by $\Phi = \frac{U}{q}$, q being either q_E or q_B,

$$\Phi_{ee} = \frac{M}{r}, \quad (9)$$

$$\Phi_{bb} = -\frac{2M}{r}, \quad (10)$$

$$\Phi_{eb} = \frac{3}{2}\frac{M}{r}, \quad (11)$$

$$\Phi_{be} = -\frac{3M}{r}. \quad (12)$$

We note that $\Phi_{resultant} = -\frac{5}{2}\frac{M}{r} \neq \Phi_{Newtonian}$ (13)

Hence, the fields ($\vec{E}$and Φ) are color sensitive.

3. Maxwellian Equations

We now extend our theory to Gravitoelectromagnetism. First of all, we note that to every $\vec{E}$ field, there is a corresponding $\vec{B}$ field, (magnetic field). The Maxwellian equations for the four fields $\vec{E}^{ij}$ are as follows: (restoring G);

$$\vec{\nabla}.\vec{E}_{gm}^{ij} = -4\pi G\rho_g, \tag{14}$$

$$\vec{\nabla}.\vec{B}_{gm}^{ij} = 0, \tag{15}$$

$$\vec{\nabla}\times\vec{E}_{gm}^{ij} = -\frac{\partial}{\partial t}\vec{B}_{gm}^{ij}, \tag{16}$$

$$\vec{\nabla}\times\vec{B}_{gm}^{ij} = \frac{1}{c^2}\frac{\partial}{\partial t}\vec{E}_{gm}^{ij}, \tag{17}$$

where mass current $\vec{j} = 0$ is used [5].

In free space, the wave equations are ($\rho = 0, \vec{j} = 0$)

$$\nabla^2\vec{E}_{gm}^{ij} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}_{gm}^{ij}, \tag{18}$$

$$\nabla^2\vec{B}_{gm}^{ij} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{B}_{gm}^{ij}. \tag{19}$$

Solutions to these equations can be written as

$$\vec{E}_{gm}^{\alpha}(\vec{r},t) = \vec{E}_0^{ij} exp\{i(\vec{k}.\vec{r} - \omega t)\}, \tag{20}$$

$$\vec{B}_{gm}^{\alpha}(\vec{r},t) = \vec{B}_0^{ij} exp\{i(\vec{k}.\vec{r} - \omega t)\}, \tag{21}$$

where α≡1, 2, 3, 4, and $i = e, b$ and $j = e, b$.

Note that from equations (14) to equations (17), each equation yields four equations corresponding to four color fields.

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Moreover, $\vec{\nabla}.\vec{E}_{gm}^{\alpha} = \hat{k}.\vec{E}_{gm}^{\alpha} = 0$ (The fields are transverse). (22)

And

$$\vec{\nabla}.\vec{B}_{gm}^{\alpha} = \hat{k}.\vec{B}_{gm}^{\alpha} = 0, \tag{23}$$

where, $\hat{k}$ is the wave vector.

Also, we have analogously to electrodynamics

$$\vec{B}_{gm}^{\alpha} = \frac{1}{c}\hat{k}\times\vec{E}_{gm}^{\alpha}. \tag{24}$$

That is, $\vec{B}$ fields are orthogonal to $\vec{E}$ fields. $\vec{E}_{gm}^{\alpha}$ are known and $\vec{B}_{gm}^{\alpha}$ can be found from $\vec{E}_{gm}^{\alpha}$ and equation (24).

The Lorentz force in this scenario is

$$\vec{F}_{gm}^{\alpha} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}(\vec{E}_{gm}^{\alpha} + \frac{\vec{v}}{c}\times\sqrt{g_{ij}}\vec{B}_{gm}^{\alpha}). \tag{25}$$

where $g_{ij} = (1 + \frac{2\Phi_{ij}}{c^2})$

4. Circularly polarized GEM waves and their influence on a test particle

We shift the coordinate system from the source of the fields to the plane where the test particle is at the xy plane with $z = 0$. The gravitational wave coming from the source of field along the z-direction and interacts with the particle through four forces given by equations (5).

Now we use circularly polarized color fields. Each of the fields $\vec{E}_{gm}^{\alpha}(z,t) = \vec{E}_0^{ij} exp\{i(k.z - \omega t)\}$ can be either left circular or right circular.

In case of right circular polarization we have four fields as

$$\vec{E}_1^R = \frac{M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} + \sin(kz - \omega t)\hat{y}\}, \quad (26)$$

$$\vec{E}_2^R = -\frac{2M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} + \sin(kz - \omega t)\hat{y}\}, \quad (27)$$

$$\vec{E}_3^R = \frac{3}{2}\frac{M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} + \sin(kz - \omega t)\hat{y}\}, \quad (28)$$

$$\vec{E}_4^R = -\frac{3M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} + \sin(kz - \omega t)\hat{y}\}. \quad (29)$$

The left circularly polarized waves are

$$\vec{E}_1^L = \frac{M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} - \sin(kz - \omega t)\hat{y}\}, \quad (30)$$

$$\vec{E}_2^L = -\frac{2M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} - \sin(kz - \omega t)\hat{y}\}, \quad (31)$$

$$\vec{E}_3^L = \frac{3}{2}\frac{M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} - \sin(kz - \omega t)\hat{y}\}, \quad (32)$$

$$\vec{E}_4^L = -\frac{3M}{r_0^2}\{\cos(kz - \omega t)\,\hat{x} - \sin(kz - \omega t)\hat{y}\}. \quad (33)$$

Referring again to the source we use , $\vec{r}_0 = z_0\hat{z}$ and location of the particle is $\vec{r} = x\hat{x} + y\hat{y} + z_0\hat{z}$.

Equation of motion of the particle is given by Lorentz equation, but without the second term, since we assume initial velocity of the particle ≈ 0. Hence for the test particle of mass m_0, the equation of motion is (with right circularly polarized incoming wave)

$$m_0\ddot{\vec{r}} = \vec{F} = (m_0)\vec{E}_1^R + (-2m_0)\vec{E}_2^R + (-2m_0)\vec{E}_3^R + m_0\vec{E}_4^R. \quad (34)$$

And $m_0\ddot{\vec{r}} = \vec{F} = (m_0)\vec{E}_1^L + (-2m_0)\vec{E}_2^L + (-2m_0)\vec{E}_3^L + m_0\vec{E}_4^L$, (35)

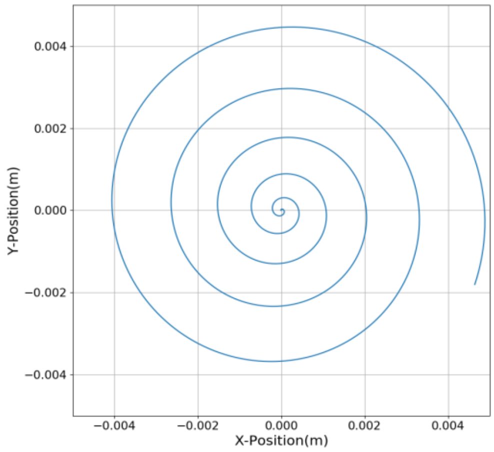
when the incoming wave is left circularly polarized.

We replace all z in equations (26) to (33) by z_0. Moreover, in equations (34) and (35), the left hand side contains m_0, rather than m_0 and $2m_0$, because effectively the mass of the test body is m_0.

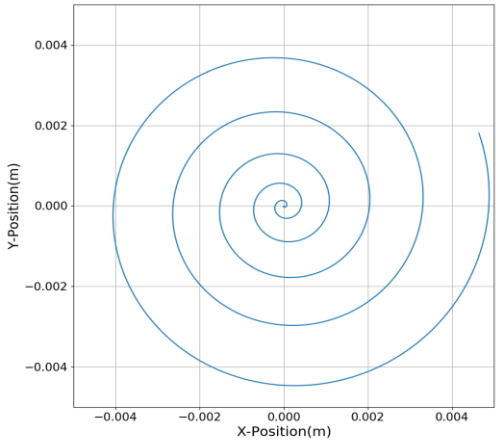
4.1. Procedure for obtaining trajectory:

To obtain the trajectory we use equations (34) and (35). Let the initial position of the particle is $(x, y) = (0,0)$ at time $t = 0$. We consider a time interval Δt, which is very small, so that the particle acceleration a remains constant. We also consider the particle motion which is initially at rest i.e.; $u_0 = 0$ for figure (1).

Now according to the well-known classical formulae for displacement, $x = x_0 + u_0 t + \frac{1}{2}at^2$ for uniform acceleration. For x-axis,$x_0 = 0, u_{0x} = 0$, so equation $x = x_0 + u_0 t + \frac{1}{2}at^2$ turns to $x_1 = \frac{1}{2}a_x(\Delta t)^2$, $x_2 = \frac{1}{2}a_x(2\Delta t)^2$, $x_3 = \frac{1}{2}a_x(3\Delta t)^2$, etc. where $a_x = -\cos(kz_0 - \omega t), \omega = 2\pi, kz_0 = 1, total\ time, t = 0.1s, total\ step, n = 1000,\ \Delta t = t/n$. Thus we can obtain the positions of the particle in x- coordinate consecutively. Similarly, for y-coordinate, $y_0 = 0, u_{0y} = 0$, so equation $y = y_0 + u_0 t + \frac{1}{2}at^2$ turns to $y_1 = \frac{1}{2}a_y(\Delta t)^2$, $y_2 = \frac{1}{2}a_y(2\Delta t)^2$, $y_3 = \frac{1}{2}a_y(3\Delta t)^2$ etc. where $a_y = \sin(kz_0 - \omega t)\,, \omega = 2\pi, kz_0 = 1$, $total\ time, t = 0.1s,\ total\ step, n = 1000,\ \Delta t = t/n$. Thus we can obtain the positions of the particle in y-coordinate consecutively. Here every point of xy plane is calculated for different acceleration in different time. Plotting these values in xy coordinate, we observe the trajectory spreads from the origin and is unbound as shown in figure 1(a) and 1(b) which is analogous to P.M. Zhang's [7] result. Zhang et al. worked out the trajectory of a particle in both plane periodic gravitational wave background and circularly polarized wave background. But in their calculation no mention is found for the two circular waves. Moreover, they have shown that in weak gravitational wave background the particle remains bounded into a toroidal region and in strong field the particle trajectory gets unbounded. In view of their result, we can say that our result correspond to strong gravitational wave field in both left and right circular wave background.



1(a)



1(b)

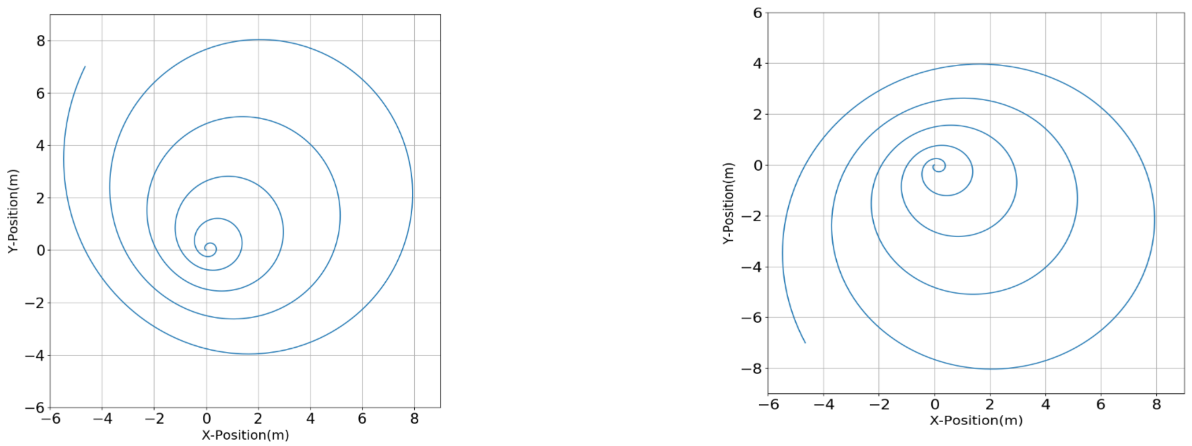
Figure 1. Trajectory of a particle initially at rest at $x = y = 0$(initial values of x and y). Fig. 1(a) shows the right circular displacement and Fig. 1(b) shows the left circular displacement with time with constant accelerationdue to gravitational wave.

4.2. Another procedure for obtaining trajectory:

We calculate the particle position in a different way for another concern depicted in figure (2). Let the initial position of the particle is $(x, y) = (0,0)$ at time $t = 0$. We consider a time interval Δt, which is very small, so that the particle acceleration a remains constant. We also consider the particle motion which is initially at rest i.e.;$u_0 = 0$. According to the well-known classical formulas for velocity and displacement $u = u_0 + at$ and $x = x_0 + u_0 t + \frac{1}{2}at^2$ respectively for uniform acceleration, for x −axis, the position of *the* particle be $x_1 = x_0 + u_{1x}(\Delta t) + \frac{1}{2}a_x(\Delta t)^2$ where, $u_{1x} = u_{0x} + a_x\,(\Delta t)$, $a_x = -\cos(kz_0 - \omega t), \omega = 2\pi, kz_0 = 1$, $total\ time, t = 0.1s,\ total\ step, n = 1000,\ \Delta t = t/n$ and $x_2 = x_1 + u_{2x}(2\Delta t) + \frac{1}{2}a_x(2\Delta t)^2$ where, $u_{2x} = u_{1x} + a_x\,(2\Delta t)$, and, $x_3 = x_2 + u_{3x}(3\Delta t) + \frac{1}{2}a_x(3\Delta t)^2$ where, $u_{3x} = u_{2x} + a_x\,(3\Delta t)$, and so on . For y −axis, the position of the particle be $y_1 = y_0 + u_{1y}(\Delta t) + \frac{1}{2}a_y(\Delta t)^2$ where, $u_{1y} = u_{0y} + a_y\,(\Delta t)$, $a_y =$

$\sin(kz_0 - \omega t)$, $\omega = 2\pi, kz_0 = 1, total\ time, t = 0.1s, total\ step, n = 1000$, $\Delta t = t/n$, and $y_2 = y_1 + u_{2y}(2\Delta t) + \frac{1}{2}a_y(2\Delta t)^2$ where $u_{2y} = u_{1y} + a_y\,(2\Delta t)$, and, $y_3 = y_2 + u_{3y}(3\Delta t) + \frac{1}{2}a_y(3\Delta t)^2$

where $u_{3y} = u_{2y} + a_y\,(3\Delta t)$, and etc. Here the acceleration of the particle acted longer than the previous one in time. Plotting these values in xy coordinate, we observe the elliptically polarized light for left and right circular oscillation using equations (34) and (35) as shown in figure 2(a) and 2(b).



2(a) 2(b)

Figure 2. Trajectory of a particle initially at rest at $x = y = 0$(initial values of x and y). Fig. 2(a) shows the right circular displacement and Fig. 2(b) shows the left circular displacement with time in successive acceleration of a gravitational wave.

5. Discussion and Conclusion

Using the theory of Gravitoelectromagnetism for slowly varying gravitational sources where the electrodynamical formulae are applicable, we redefine the Gravitoelectromagnetism using four different fields. We have shown that these fields give correctly the Newtonian force and potential energy. But they do not add up to give field and potential of Newtonian theory. We then incorporate Maxwell's electromagnetic equations with four different color fields. We show the influence of a gravitational wave on a single particle. It is shown that the trajectory of the particle on which gravitational wave acts, spreads outside and it becomes unbound that indicates its higher energy. Therefore, the particle absorbs energy from the incoming wave. It is also shown for a particular iteration process of computation; this trajectory of the particle for left and right circular polarization are elliptical. If the wave impinges obliquely, then the trajectory would be ellipsoid. That is, it will absorb radiation with quadrupole moment, i.e., particle of gravitational wave would be of spin=2. These are signs of the incoming wave as if they are like general relativistic gravitational wave. Hence, with four independent fields, we have rediscovered some of Newtonian and some of Einsteinian gravity. This work, thus, heralds a new paradigm of gravitation theory.

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